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MATHEMATICAL MODELING OF BANDITRY INCORPORATING REPENTANT BANDITS AND GOVERNMENT EFFORTS

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ABSTRACT

Banditry is one of the major global societal problem. In view of this, a mathematical model of banditry was developed incorporating repentant bandits and government efforts. The model is constructed to control banditry activities in the society. The population is divided in to eight (8) compartments: S1 ,S2,B,C,F1,F2,R and G. Furthermore, the effective reproduction number RC was calculated. For the control parameters , RC < 1 the sociological implication of RC < 1 is banditry is nearly eradicated, and for control parameters then RC,>1 meaning the number of bandits will persist to endemic. A locally asymptotically stable of bandit-free and bandits-present equilibrium where established, the global stability was established using Lyapunov theorem. The finding shows that effort of government in eliminating and rehabilitating of bandits are best way of tackling banditry activities.

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1.INTRODUCTION

Banditry is a type of organized crime committee by outlaws typically involving the threat or use of violence. A person who engages in banditry is known as a bandit, they commit crimes such as extortion, kidnapping, robbery, murder, rapping, etc. either as an individual or group (Rufai, 2021).
Human existence and continuity have faced threats from a variety of sources. In fact, it is characterized by self and societal anarchy and chaos, which might result from biases, hatred, and violent ideologies. If one bears all of these features in mind, some of the complexities of dealing with terrorist organizations become immediately apparent. Any attempt to combat terrorism is usually two-edged: to end existing ones and to forestall new ones from springing up (Okoye *et al*., 2020).
A mathematical model that includes the effect of government efforts against ideologists. Government efforts such as counter-extremism using technology serve as an effective means to control the extremist's activities. (Hoffman, 2006) studied new techniques for interactions between government resources and terrorist activities for the optimization of its resources. It has been widely noted that the government efforts can curb the activities of the extremists by using modern technology leading to an increase in popular support of the government and preventing people's support of the extremists.
Although banditry and terrorism are social menace, however mathematicians around the world contributed toward eradicating their activities through mathematical modeling. Such as Momo et al (2023) make an effort to expand corpus of research that applies mathematical models to tackle difficult societal problems, particularly to shed light on the inner workings of banditry. The goal of this research is to solve the enigmas surrounding banditry and provide information that can guide better preventative and control measures in this vibrant and dynamic environment. This study aims to go beyond the limitations of conventional analysis by employing a quantitative, data driven methodology and offering fresh tools to assist academics, law enforcement, and politicians in addressing the intricate issues that banditry poses. It seeks to provide a clearer understanding of the banditry environment by exploring mathematical modeling, empowering us to take more targeted and knowledgeable action.

Aniayam *et al*., (2018) constructs a simple mathematical model on kidnapping by integrating the concept of deradicalizing and rehabilitation of kidnappers in a system of ordinary differential equations describing the evolution and propagation of kidnapping as a crime in human society, their study accounts for the interaction between kidnappers and vulnerable humans leading to their abduction for the main purpose of ransom payment. They also established the crime propagation number, Cpn in which a $Cpn<1$ guarantees a kidnap free state that was locally and globally asymptotically stable. The analysis shows that increasing the rehabilitation rate of kidnappers was a better and more effective way of ensuring a kidnapping free society. Julius *et al.,* (2021) also studied the dynamics of terrorism in contemporary society for effective management.Okoye *et al*., (2020) investigates how to improve the understanding of the policymakers on terrorism mitigation. They consider a place with total population N at time t under the attack of terrorists. Assume that the total population N is partitioned into three distinct classes namely: Susceptible, Terrorists, and Deserted classes denoted, respectively, by X, Y, and Z. The authors adopted a mathematical modeling and theoretical designed.
Adamu and Ibrahim (2020) developed a model to control the spread of terrorist ideologies in society and suitable to described a terrorist group. In this research, the populations of interest are Susceptible S(t), Moderate I(t), Terrorist T(t) Terrorist soldiers TS(t), Terrorist leaders TL(t) and Detention facility QT (t). Susceptible individuals are recruited into the population by input rate Λ . When susceptible individuals have contact(s) with some terrorists, the probability of embracing the ideology β_1 but embracing the ideology did not make them extremist except with a proportion of β_2 chance for which

the individual move to terrorist group. The terrorist group is further split into foot soldiers TS(t) and leaders TL(t), alternatively to become extremist in terrorism this depends on the moderate individual increase in relation with either terrorist leader or foot soldiers. The finding shows that the military/dialogue strategies were to be used while military strategies alone should not be used if the number of terrorists was below a certain basic reproduction number. Hussain (2019) Investigate the dynamical behavior model on the network of militants. Akanni and Abidemi (2023) studied the global stability of illicit drug used spread dynamics with banditry compartment using a dynamical system theory approach. Illicit drug used and banditry reproduction number was evaluated analytically, which measures the potential spread of the illicit drug used and banditry in the population. They reported that system exhibits supercritical bifurcation property, also local stability of an illicit drug and banditry-present equilibrium exist and it was unique. Thus, the illicit drug and banditry-free and illicit drug and banditry-present equilibria were shown to be global asymptotically stable, it was achieved by construction of suitable Lyapunov functions. Suggested control measures to use in curtail the menace of the illicit drug use and banditry were recommends. Lawal et al (2023) provide a quantitative framework for limiting the armed banditry epidemic in Nigeria. Five ordinary differential equations make up the model, which accounts for the dynamics of the bandits B(t), informants I(t), recovered people R(t), and susceptible individuals S(t) and E(t). The effectiveness of two time-dependent controls job creation and initiatives to render armed banditry unprofitable in lowering the demographic profile of informants and bandits is examined by the authors. The Forward-Backward Sweep Method of the fourth-order Runge-Kutta scheme is used by the authors to simulate and define the optimal control model utilizing the Pontryagin Maximum Principle (PMP). Recently Tasiu et al (2024a) developed a mathematical model on dynamics of banditry: mathematical modelling approach, the authors incorporate freed individuals, individuals undergoing detention and government intervention. Thus this work tend to develop a mathematical model of banditry by considering repentant bandits, rehabilitation and government efforts.

2. MODEL FORMULATION

2.1 Model Description. In this work, the population is divided in to : Non-recruitable susceptible (S_1), the recruitable susceptible (S_2), the bandits B, the captives C, individuals undergoing rehabilitation R, the repented bandits (F_1), the freed individuals (F_2) and efforts of government to engaged the bandits. The population of Nonrecruitable susceptible, S_1 are generated from daily recruitment through birth or by immigration at the rate $\Lambda\theta$. They decrease due to kidnapping and natural death .The population of Recruitable susceptible, S_2 is also generated by daily recruitment at $\Lambda(1-\theta)$ and diminishes by movement to banditry class at the rate of λ_1 and natural death at the rate of μ.

The Bandits class (B) is generated through influx from recruitable susceptible class at the rate of λ_1 and diminished by natural death or eliminated by government at the rates of μ and δ_3 and also progression to rehabilitation (R) at the rate of j. While,The Repentant bandits, F_1 are those that are freed from rehabilitation at the rate of γ_3 and decreases by natural death at μ.

The Freed class (F_2) That is those that regained freedom from captivity by either paying ransom or government efforts is generated by progression from captive class at the rate of $(\gamma_1 + \gamma_2)$ finally natural death occurs in all classes at a rate μ . The schematic diagram is represented in figure 1

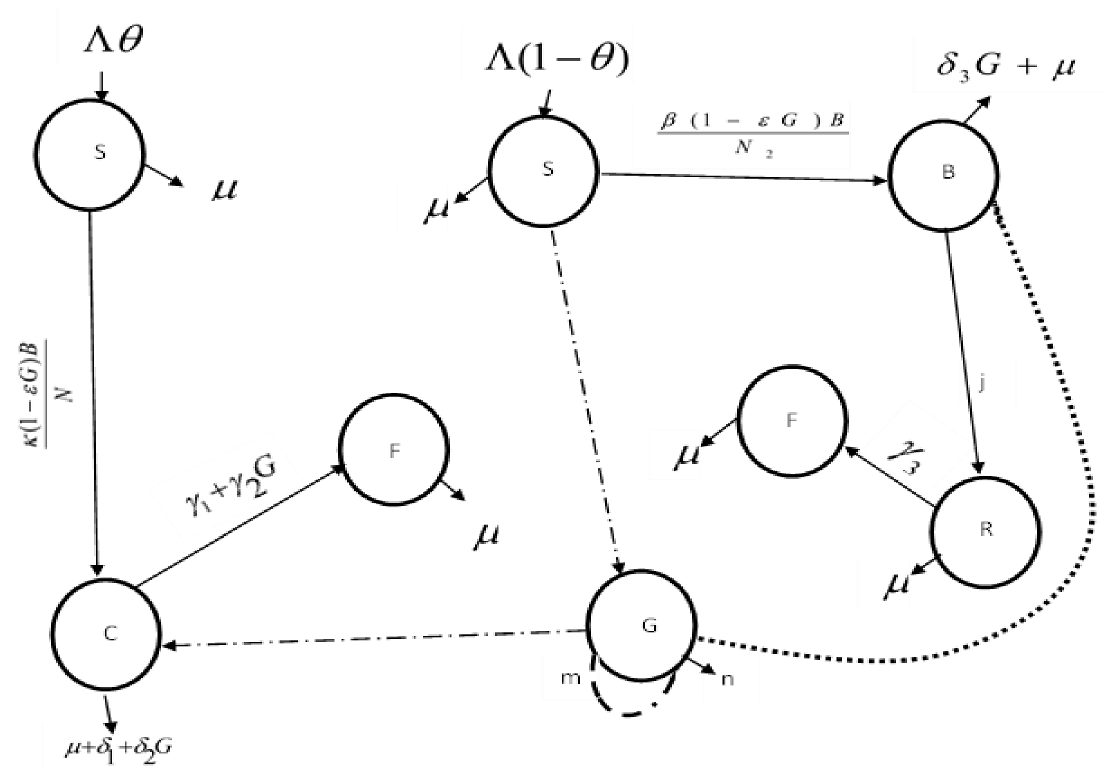


Figure 1: Schematic diagram of bandits model

The corresponding mathematical representation of the schematic diagram in figure .1 is given by subsequent deterministic system of non-linear equations (1)

$$
\left.
\begin{aligned}
\frac{dS_1}{dt} &= \Lambda\theta - \frac{\kappa(1-\varepsilon_1 G)BS_1}{N} - \mu S_1 \\
\frac{dS_2}{dt} &= \Lambda(1-\theta) - \frac{\beta(1-\varepsilon G)B}{N_2}S_2 - \mu S_2 \\
\frac{dB}{dt} &= \frac{\beta(1-\varepsilon G)B}{N_2}S_2 - (j+\mu+\delta_3 G)B \\
\frac{dC}{dt} &= \frac{\kappa(1-\varepsilon_1 G)BS_1}{N} - (\gamma_1+\mu+\delta_1 C) - (\gamma_2+\delta_1)GC \\
\frac{dF_1}{dt} &= (\gamma_1+\gamma_2 G)C - \mu F_1 \\
\frac{dF_2}{dt} &= \gamma_3 R - \mu F_2 \\
\frac{dR}{dt} &= jB - (\gamma_3+\mu)R \\
\frac{dG}{dt} &= m - nG
\end{aligned}
\right\} \quad . \qquad (1)
$$

where

$$N_1 = S_1 + C + F_1.$$

$$N_2 = S_2 + B + R + F_2.$$

$$N = N_1 + N_2.$$

and let $\lambda = \frac{\beta(1-\varepsilon_2 G)B}{N_2}$, (2)

so that

$$\frac{dN_1}{dt} = \Lambda\theta - \mu N_1 - (\delta_1 + \delta_2 G)C$$

$$\frac{dN_2}{dt} = \Lambda(1-\theta) - \mu N_2 - \delta_3 GB$$

$$\frac{dN}{dt} = \Lambda - \mu N - (\delta_1 C + \delta_2 GC + \delta_3 GB)$$

$$\frac{dN_2}{dt} = \Lambda(1-\theta) - \mu N_2 - \delta_3 GB.$$

The model (1) is sociologically and mathematically well-posed in the domain,

$$\Omega = \left\{ \begin{pmatrix} S_1 \\ S_2 \\ B \\ C \\ R \\ F_1 \\ F_2 \\ G \end{pmatrix} \in \mathbb{R}_+^8 \;\middle|\; \begin{matrix} S_2 \geq 0, \\ S_1 \geq 0, \\ B \geq 0, \\ C \geq 0, \\ R \geq 0, \\ F_1 \geq 0, \\ F_2 \geq 0, \\ G \geq 0, \\ S_1 + S_2 + B + C + R + F_1 + F_2 + G \leq N \end{matrix} \right\}. \quad (3)$$

This domain, Ω, is sociologically valid as the sub-populations, $S_1, S_2, B, C, R, F_1, F_2$, and G, are all non-negative and have sum less than or equal the total population, N.

3. Basic Properties of banditry model

3.1 Invariant Region: The population size can be determined by differential equation of the model formulated.

$$\frac{dN}{dt} = \frac{dS_1}{dt} + \frac{dS_2}{dt} + \frac{dB}{dt} + \frac{dC}{dt} + \frac{dR}{dt} + \frac{dF_1}{dt} + \frac{dF_2}{dt}. \quad (4)$$

This can be reduced to

$$\frac{dN}{dt} = \Lambda - \mu N - (\delta_2 G + \delta_1)C - \delta_3 G B,$$

Such that

$$\frac{dN}{dt} \leq \Lambda - \mu N. \quad (5)$$

Since $N = S_1 + S_2 + B + C + R + F_1 + F_2 + G$ and equation (5) resolved to linear differential equations of the form:

$$\frac{dN}{dt} + \mu N \le \Lambda. \tag{6}$$

Theorem 1: The solution of the system of model (1) is feasible for $t < 0$ if they are in invariant region Ω. (Diekmann & Heesterbeek, 1990).

Proof: let $(S_1, S_2, B, C, R, F_1, F_2, G) \in R^8$ be any solution of the system with non-negative initial conditions using integrating factor.

$$I.F = e^{\int pdt} = e^{\mu t} + c = e^{\mu t}.e^{c} = Ae^{\mu t},$$

$$Ae^{\mu t}\frac{dN}{dt} = \int Ae^{\mu t}\Lambda \quad \text{Such that}$$

$$N(t) = \frac{\Lambda}{\mu} + Ce^{-\mu t}, \tag{7}$$

At $t = 0$ the initial population will become

$$N(0) = \frac{\Lambda}{\mu} + C \quad \text{, where C is constant} \tag{8}$$

Simplifying we have,

$$C = N_O - \frac{\Lambda}{\mu}.$$

By substitution

$$N(t) = \frac{\Lambda}{\mu} + \left(N_0 - \frac{\Lambda}{\mu}\right)e^{-\mu t},$$

$$N(0) \le \frac{\Lambda}{\mu}.$$

at $t \to \infty$ in (7) the human population N approaches $C = \frac{\Lambda}{\mu}$ i.e $N \to C$

where $C = \frac{\Lambda}{\mu}$ is the caring capacity.

As a result, all possible solution set for the model system's population in (1) entered the region.

.

$$\Omega = \left\{ S_1, S_2, B, C, R, F_1, F_2, G \in R_+^8 : S_1, S_{2,} B, C, R, F_1, F_2, G \ge 0 \therefore N \le \frac{\Lambda}{\mu} \right\}.$$

Hence its positively invariant set under the flow induced by the model (1) hence the model is mathematically well posed in the domain. Furthermore, the usual existence, continuity and uniqueness of results holds for the system.

3.2 Existence of Bandits-free Equilibrium State $\left(E^0\right)$

At Bandits-free equilibrium state there is no bandits. Thus, all the infected classes will be zero and the entire population will comprise of recruitable susceptible and non-recruitable susceptible individuals. By solving the model equations as in Driessche & Watmough (2002).

$$\begin{pmatrix} S_1 \\ S_2 \\ B \\ C \\ R \\ F_1 \\ F_2 \\ G \\ N_2 \\ N \\ N_1 \end{pmatrix} = \begin{pmatrix} \frac{\Lambda\theta}{\mu} \\ \frac{\Lambda(1-\theta)}{\mu} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{m}{n} \\ \frac{\Lambda(1-\theta)}{\mu} \\ \frac{\Lambda}{\mu} \\ \frac{\Lambda\theta}{\mu} \end{pmatrix}, \tag{9}$$

By simplification and substitution of (2) we arrived at

$$\lambda^* = 0 , \tag{10}$$

$$\text{Or } \begin{Bmatrix} n(j+\mu)\lambda^2 + \left(\mu n(j+\mu) + \mu(jn+\mu n+\delta_3 m) - \beta(n-\varepsilon_1 m)\mu\right)\lambda + \\ \mu^2(jn+\mu n+\delta_3 m) - \beta(n-\varepsilon_1 m)\mu^2 \end{Bmatrix} = 0 . \tag{11}$$

3.3 Local Stability of Bandits-free Equilibrium and Effective Reproduction number

We established the stability of bandits-free equilibrium by using next generation matrix developed by Diekmann & Heesterbeek (1990) and subsequently analysed by Drieessche and Watmough (2002) as well as Tasiu et al (2024), we obtain the effective reproduction number R_C of the model (1).which is the spectral radius (ρ) of next generation matrix FV^{-1}. The matrix F for the new infection terms and V for the transmission terms are obtained from the infected compartments (R, B and F_2) given at bandits-free equilibrium and given, respectively by:

$$F = \begin{pmatrix} \frac{\beta(n-\varepsilon_1 m)}{n} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

$$V=\begin{pmatrix} \frac{(jn+\mu n+\delta_3 m)}{n} & 0 & 0 \\ \frac{-\kappa(n-\varepsilon_1 m)\mu}{\Lambda n} & \frac{n(\gamma_1+\mu+\delta_1)+(\gamma_2+\delta_2)m}{n} & 0 \\ -j & 0 & (\gamma_3+\mu) \end{pmatrix}, \tag{13}$$

$$V^{-1}=\begin{pmatrix} \frac{n}{(jn+\mu n+\delta_3 m)} & 0 & 0 \\ \frac{\kappa(n-\varepsilon_1 m)n\mu}{\Lambda(jn+\mu n+\delta_3 m)\left[n(\gamma_1+\mu+\delta_1)+(\gamma_2+\delta_2)m\right]} & \frac{n}{n(\gamma_1+\mu+\delta_1)+(\gamma_2+\delta_2)m} & 0 \\ \frac{jn}{(jn+\mu n+\delta_3 m)(\gamma_3+\mu)} & 0 & \frac{1}{(\gamma_3+\mu)} \end{pmatrix}, \tag{14}$$

Multiflyig (12) and (14) yields to:

$$FV^{-1}=\begin{pmatrix} \frac{\beta(n-\varepsilon_1 m)}{(jn+\mu n+\delta_3 m)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{15}$$

Hence, the reproduction number is obtain as $R_C=\frac{\beta(n-\varepsilon_1 m)}{(jn+\mu n+\delta_3 m)}$. (16)

Thus, we will established that the bandit-free equilibrium of the model (1) is locally asymptotically stable if $R_C<1$.

Effective Reproduction number R_C is a measurement of the potential for the spread of bandits in a population. Mathematically, R_C is a threshold parameter for the stability of bandits -free equilibrium and is related to the peak and final size of an epidemic. Its defined as the expected number of secondary cases of infection which would occur due to primary case in a completely susceptible population Okuonghae (2017).

3.4. Existence of Local Stability of Bandits Free-Equilibrium Point

The asymptotic stability of equilibrium for a non-linear differential equation system is established using the Routh-Hurwitz criteria. Asymptotic stability is implied by the Routh-Hurwitz criteria, which give the necessary and sufficient conditions for all roots of the characteristic polynomial to contain negative components Merkin (1997). The following information on the equilibrium's local stability can be found in the Jacobian matrix:

$$J\left(E^{0}\right)=\begin{pmatrix} -\mu & 0 & -k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu & -k_2 & 0 & 0 & 0 & 0 & k_3 \\ 0 & 0 & k_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_5 & -k_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_7 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_3 & 0 & -\mu & 0 \\ 0 & 0 & j & 0 & -k_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n \end{pmatrix}, \qquad (17).$$

i.e.

$$\left|J\left(E^{0}-\lambda I\right)\right|=\begin{pmatrix} -(\mu+\lambda) & 0 & -k1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\mu+\lambda) & -k2 & 0 & 0 & 0 & 0 & k3 \\ 0 & 0 & k4-\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k5 & -(k6+\lambda) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k7 & -\lambda & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_3 & -\lambda & -\mu & 0 \\ 0 & 0 & j & 0 & -k8 & 0 & -\lambda & 0 \\ 0 & 0 & & 0 & 0 & 0 & 0 & -(n+\lambda) \end{pmatrix}, \qquad (18).$$

Therefore, the bandits-free equilibrium E^0 is LAS when there is no bandits. It follows that the characteristic equation of (18) computed is given by solving the determinant with maple16 software which yields to,

$$(k4-\lambda)\,(-\mu-\lambda)^2\,(-n-\lambda)\,(-k6-\lambda)\left(-\lambda^3-\gamma 3\,\mu\,\lambda-k8\,\mu^2\right) \quad . \qquad (19)$$

where,

$$k1=-\kappa\left(\frac{n-\varepsilon m}{n}\right),\ k2=-\beta\left(\frac{n-\varepsilon m}{n}\right),\ k3=-\beta(1-\varepsilon),\ k4=\beta(n-\varepsilon m-jn+\mu n+\delta_3 m)\frac{\mu}{\Lambda(1-\theta)}$$

$$k5=-\kappa\left(\frac{n-\varepsilon m}{n}\right),\ k6=-\left(\frac{\delta_1 n+\gamma_2 m+\delta_2 m}{n}\right),\ k7=\left(\frac{\gamma_1 n+\gamma_2 m}{n}\right),\ k8=-(\gamma_3+\mu).$$

simplifying (10) gives,

$$\lambda^8 + (n - k4 + k6)\lambda^7 + (2n\mu + 2\mu^2 - 2\mu k4 + nk4 + nk6 - k4 + \gamma_3\mu)\lambda^6$$
$$+\left(2n\mu k4 - \mu^2 k4 + n\mu^2 + 2n\mu k6 - 2\mu k4k6 + 2\mu^2 k6 + nk4k6 - \mu k4\gamma_3 + n\mu\gamma_3 + \mu\gamma_3 k6 + k8\mu^2\right)\lambda^5$$
$$+\begin{pmatrix} 2nk4k6\mu - nk4\mu^2 - k4k6\mu^2 + n\mu^2 - 2\mu^2 k4\gamma_3 + \mu k4n\gamma_3 + 2\mu^3\gamma_3 n + n\mu\gamma_3 k6 - \mu\gamma_3 k4k6 + \\ nk8\mu^2 - k4k8\mu^2 + k6k8\mu^2 \end{pmatrix}\lambda^4$$
$$+\begin{pmatrix} 2\mu^2\gamma_3 k4n - k4\mu^3\gamma_3 + n\mu^3\gamma_3 + 2\mu k4k6\gamma_3 + 2n\mu^2 k6\gamma_3 + 2\mu^3 k6\gamma_3 \\ +\mu\gamma_3 k4k6 + nk4k8\mu^2 + 2k8\mu^4 + 2nk8\mu^3 + 2k4k8\mu^3 + nk6k8\mu^2 - k4k6k8\mu^2 \end{pmatrix}\lambda^3 \tag{20}$$
$$+\left(2n\gamma_3\mu k4k6 - 2\mu^4 k6k8 + 2nk6k8\mu^3 - 2k4k6k8\mu^3\right)\lambda^2$$
$$+\left(2nk4k6k8\mu^3 - \mu^3 nk4k6 - nk4k8\mu^4 - k4k6k8\mu^4 + nk6k8\mu^4\right)\lambda + \left(-nk4k6\mu^2 - nk4k6k8\mu^6\right).$$

Collect the coefficient of the eigenvalues λ and characteristics gives

$$\lambda^8 + a_7\lambda^7 + a_6\lambda^6 + a_5\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0. \tag{21}$$

where

$$a = 1.$$
$$a_7 = -(n + k4 - k6).$$
$$a_6 = -(2n\mu - 2\mu^2 + 2\mu k4 - nk4 - nk6 + k4 - \gamma_3\mu).$$
$$a_5 = \left(2n\mu k4 - \mu^2 k4 + n\mu^2 + 2n\mu k6 - 2\mu k4k6 + 2\mu^2 k6 + nk4k6 - \mu k4\gamma_3 + n\mu\gamma_3 + \mu\gamma_3 k6 + k8\mu^2\right).$$
$$a_4 = (2nk4k6\mu - nk4\mu^2 - k4k6\mu^2 + n\mu^2 - 2\mu^2 k4\gamma_3 + \mu k4n\gamma_3 + 2\mu^3\gamma_3 n + n\mu\gamma_3 k6 - \mu\gamma_3 k4k6 + nk8\mu^2 - k4k8\mu^2 + k6k8\mu^2).$$
$$a_3 = \begin{pmatrix} 2\mu^2\gamma_3 k4n - k4\mu^3\gamma_3 + n\mu^3\gamma_3 + 2\mu k4k6\gamma_3 + 2n\mu^2 k6\gamma_3 + 2\mu^3 k6\gamma_3 \\ +\mu\gamma_3 k4k6 + nk4k8\mu^2 + 2k8\mu^4 + 2nk8\mu^3 + 2k4k8\mu^3 + nk6k8\mu^2 - k4k6k8\mu^2 \end{pmatrix}.$$
$$a_2 = \left(2n\gamma_3\mu k4k6 - 2\mu^4 k6k8 + 2nk6k8\mu^3 - 2k4k6k8\mu^3\right).$$
$$a_1 = \left(2nk4k6k8\mu^3 - \mu^3 nk4k6 - nk4k8\mu^4 - k4k6k8\mu^4 + nk6k8\mu^4\right).$$
$$a_0 = \left(-nk4k6\mu^2 - nk4k6k8\mu^6\right). \tag{22}$$

Using the Routh-Hurwitz criterion, it can be clearly seen that all the eigenvalues have negative real part and therefore the bandit-free equilibrium is LAS since there is no bandit.

4. Global Stability of Bandits-free Equilibrium (E^0)

Theorem 3: The bandit-free equilibrium, E^0 of (1) is globally asymptotically stable (GAS) in Ω if $R_C < 1$.

Proof: We construct an appropriate Lyapunov function

Consider the Lyapunov function

$$L = aB + bR, \tag{23}$$

the derivatives along the solution of (23) is

$$L' = aB' + bR', \tag{24}$$

By substitution of B' and R' we arrived at

$$L' = a\left[\left(1-\varepsilon G\right)\lambda S_2 - \left(j+\mu+\delta_3 G\right)B\right] + b\left[jB - \left(\gamma_3+\mu\right)R\right]. \tag{25}$$

We set the coefficient of λS_2 to the numerator of numerator of R_C ignoring β and substituting G we have

$$a = n. \tag{26}$$

We set the coefficient of the R to zero

$$\left(\gamma_3+\mu\right)b = 0,$$

$$\text{then } b = 0. \tag{27}$$

Substituting (26) and (27) we have

$$L' = nB', \tag{28}$$

$$L' = n\left(j+\mu+\delta_3 G\right)B\left\{\frac{\beta\left(1-\varepsilon G\right)S_2}{N_2\left(j+\mu+\delta_3 G\right)} - 1\right\}. \tag{29}$$

Since,

$$N_2 \le N_2^{\,0}, S_2 \le S_2^{\,0}, G \le G^0 \tag{30}$$

$$L' \le \left(jn+\mu n+\delta_3 m\right)B\left\{\frac{\beta\left(n-\varepsilon m\right)}{\left(jn+\mu n+\delta_3 m\right)} - 1\right\}, \tag{31}$$

$$L' \le \left(jn+\mu n+\delta_3 m\right)B\left\{R_C - 1\right\}. \tag{32}$$

Since the model parameters are non-negative, it follows that $L \le 0$ for $R_C \le 1$, with L =0 if and only if $B = R = 0$. Hence L = is Lyapunov function in the invariant region, thus the above theorem shows that the sociological requirements of $R_C \le 1$ is sufficient condition for the elimination of banditry.

5 Existence of Endemic Equilibrium State $\left(E^{**}\right)$

The endemic equilibrium state or point is a positive steady state solution at which the bandits persist in the population. That is the coordinates should satisfy the conditions:

$$E^{**} = \left\{ \begin{pmatrix} S_1 \\ S_2 \\ B \\ C \\ R \\ F_1 \\ F_2 \\ G \end{pmatrix} \middle| \begin{array}{c} S_1 > 0, \\ S_2 > 0, \\ B > 0, \\ C > 0, \\ R > 0, \\ F_1 > 0, \\ F_2 > 0, \\ G > 0, \\ S_1 + S_2 + B + C + R + F_1 + F_2 + G \le N \end{array} \right\}, \tag{33}$$

Lemma: The endemic equilibrium state of the model (1) exists if the effective reproduction number, $R_C > 1$.

Proof: At the endemic equilibrium state, let

$$\begin{pmatrix} S_1^{**} \\ S_2^{**} \\ B^{**} \\ C^{**} \\ R^{**} \\ F_1^{**} \\ F_2^{**} \\ G^{**} \end{pmatrix} = \begin{pmatrix} S_1 \\ S_2 \\ B \\ C \\ R \\ F_1 \\ F_2 \\ G \end{pmatrix} , \tag{34}$$

Consider an arbitrary equilibrium at which equation (11) holds and let

$$A_1 = n(j+\mu) \ . \tag{35}$$

$$A_2 = \left(\beta(n-\varepsilon_1 m)\mu - \mu n(j+\mu) + \mu(jn+\mu n+\delta_3 m)\right) \ . \tag{36}$$

$$A_3 = \left\{\beta(n-\varepsilon_1 m)\mu^2 - \mu^2(jn+\mu n+\delta_3 m)\right\} \ . \tag{37}$$

Thus, (11) becomes

$$A_1\lambda^2 - A_2\lambda - A_3 = 0 \ . \tag{38}$$

Using Descartes rule of sign, there exist a unique positive equilibrium if

$$A_3 > 0 \ , \tag{39}$$

$$\left\{\beta(n-\varepsilon_1 m)\mu^2 - \mu^2(jn+\mu n+\delta_3 m)\right\} > 0 \ , \tag{40}$$

$$\frac{\beta(n-\varepsilon_1 m)}{(jn+\mu n+\delta_3 m)} > 1 \ , \tag{41}$$

$$R_C > 1 \ . \tag{42}$$

Thus, $\lambda > 0$ if $R_C > 1$. Then substituting $\lambda > 0$ at E^{**} gives

$$\left\{\begin{array}{l} S_1^{**} > 0 \\ S_2^{**} > 0 \\ B^{**} > 0 \\ C^{**} > 0 \\ R^{**} > 0 \\ F_1^{**} > 0 \\ F_2^{**} > 0 \\ G^{**} > 0 \end{array}\right\}. \tag{43}$$

Hence, the endemic equilibrium state of the model exists if $R_C > 1$.

5.1 Local stability of endemic equilibrium, E^{**}

The Routh-Hurwitz criteria is used to established asymptotic stability of equilibrium for non-linear system of differential equation. The Routh-Hurwitz criteria provide the necessary and sufficient condition for all roots of the characteristic polynomial to contain negative parts therefore entails asymptotic stability Merkin (1997). The local stability of the equilibrium may be determined from the Jacobian matrix gives

$$J\left(E^0\right)=\begin{pmatrix} -\left(\frac{\kappa\left(1-\varepsilon G^{**}\right)B^{**}}{N^{**}}+\mu\right) & 0 & -\kappa\left(1-\varepsilon G^{**}\right)S_1^{**} & 0 & 0 & 0 & 0 & \frac{\kappa\left(1-\varepsilon G^{**}\right)B^{**}S_1^{**}}{N^{**}} \\ 0 & \frac{-\beta(1-\varepsilon)B^{**}}{N_2^{**}} & \frac{-\beta\left(1-\varepsilon G^{**}\right)S_2^{**}}{N_2^{**}} & 0 & 0 & 0 & 0 & \frac{-\beta\left(1-\varepsilon G^{**}\right)S_2^{**}}{N_2^{**}} \\ 0 & \frac{\beta\left(1-\varepsilon G^{**}\right)B^{**}}{N_2^{**}} & \delta_3 G^{**} & 0 & 0 & 0 & 0 & \frac{\beta(1-\varepsilon)B^{**}S_2^{**}}{N_2^{**}}-\delta_3 B^{**} \\ \frac{\kappa\left(1-\varepsilon G^{**}\right)B^{**}}{N^{**}} & 0 & \frac{\kappa(1-\varepsilon)S_1^{**}}{N^{**}} & -\left(\delta_1+\gamma_1+\delta_2 G^{**}\right) & 0 & 0 & 0 & -\left(-\frac{\kappa\left(1-\varepsilon G^{**}\right)B^{**}S_1^{**}}{N^{**}}+\left(\gamma_2+\delta_2\right)C^{**}\right) \\ 0 & 0 & 0 & \left(\gamma_1+\gamma_2 G^{**}\right) & -\mu & 0 & 0 & \gamma_2 C^{**} \\ 0 & 0 & 0 & 0 & 0 & -\mu & \gamma_3 & 0 \\ 0 & 0 & j & 0 & 0 & 0 & -\left(\gamma_3+\mu\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n \end{pmatrix} \tag{44}$$

Using elementary row operation we have,

$$J\left(E^0\right)=\begin{pmatrix} -k_1 & 0 & k_2 & 0 & 0 & 0 & 0 & k_3 \\ 0 & -k_4 & -k_5 & 0 & 0 & 0 & 0 & k_6 \\ 0 & 0 & -k_7 & 0 & 0 & 0 & 0 & k_8 \\ 0 & 0 & 0 & -k_9 & 0 & 0 & 0 & k_{10} \\ 0 & 0 & 0 & 0 & -k_{11} & 0 & 0 & k_{12} \\ 0 & 0 & 0 & 0 & 0 & -k_{13} & \gamma_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_{14} & k_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n \end{pmatrix}, \tag{45}$$

Where

$$k_1=\left(\kappa\left(1-\varepsilon G^{**}\right)B^{**}+\mu\right),\ k_2=-\kappa\left(1-\varepsilon G^{**}\right)S_1^{**},\ k_3=\kappa(1-\varepsilon)B^{**}S_1^{**}.$$

$$k_4=\frac{\beta(1-\varepsilon)B^{**}}{N_2^{**}},\ k_5=\frac{-\beta\left(1-\varepsilon G^{**}\right)S_2^{**}}{N_2^{**}},\ k_6=\frac{-\beta\left(1-\varepsilon G^{**}\right)S_2^{**}}{N_2^{**}}.$$

$$k_7=-\left(\frac{\delta_3 G^{**}+\beta\left(1-\varepsilon G^{**}\right)S_2}{N_2^{**}}\right),\quad k_8=\beta\left(\frac{1-\varepsilon B^{**}}{N_2^{**}}\right)S_2G^{**}\ K_9=\left(\delta_1+\gamma_1+\delta_2 G^{**}\right).$$

$$k_{11}=\left(\gamma_1+\gamma_2 G^{**}\right)\quad k_{13}=\mu,\ k_{14}=\left(\gamma_3+\mu\right),\ k_{15}=-\left(\frac{j\beta(1-\varepsilon\beta)B\delta_3G^{**}S_2\delta_3G^{**}}{\delta_3G^{**}}\right).$$

Theorem : Endemic equilibrium is locally asymptotically stable if $R_C>1$,

Proof:

$$\lambda_1=-\left(\kappa\left(1-\varepsilon G^{**}\right)B^{**}+\mu\right)<0,\ \lambda_2=-\frac{\beta(1-\varepsilon G)^{**}B^{**}}{N_2^{**}}<0,\ \lambda_3=-\left(\frac{\delta_3G^{**}+\beta\left(1-\varepsilon G^{**}\right)S_2}{N_2^{**}}\right)<0,$$

$$\lambda_4=-\left(\gamma_2+\delta_1+\delta_3G^{**}\right)<0,\ \lambda_5=-\mu<0,\ \lambda_6=-\mu<0,\ \lambda_7=-\left(\gamma_3+\mu\right)<0,\ \lambda_8=-n<0.$$

The positive endemic equilibrium state of the system (1) is locally asymptotically stable (LAS) when $R_C>1$ .Ninuola (2017), Feng and Huang (2002).

The sociological implication of Theorem above is that banditry will continue to persist in the population when $R_C>1$ and the initial size of the sub-populations of the model are in the basin of attraction of the endemic state.

6. NUMERICAL VERIFICATION

In this section, we presented some numerical simulations to monitor the dynamics of model (1) for various values of reproduction number in order to validate the analytical results on stability of both bandits -free and endemic equilibria.

6.1 Parameter Estimation

Parameters values estimation is probably the most difficult section of mathematical modelling. This is because any small mis-estimation of a value may result in misappropriate output. Thus the baseline values for parameters of model (1) are presented in table 1.1 below

Table 1.1: Values for parameters of the model

S/N	Parameter	Value	Source
1	Λ	833000	Calculated
2	μ	0.017	NBS, 2023 Abdulrahman (2018)
3	β	0.1	Tasiu et al 2024
4	δ_1	0.1	Hypothetic
5	δ_2	0.01	Hypothetic
6	κ	0.022	Hypothetic
7	γ_3	0.2	Hypothetic
8	θ	0.9	Hypothetic
8	m	0.008	Calculated
9	n	0.01	Calculated
10	$\varepsilon_i, \varepsilon_2, \gamma_1, \gamma_2, j,, \delta_3$	(0-1)	Controls

The recruitment rate, Λ in to non -recruitable sub-population is assumed to be 833000 per year. The average lifespan of Nigeria is $\frac{1}{\mu}$ which gives a death removal rate (μ) of 0.017. the other model values as hypothetical values for numerical simulations, so that $N = \frac{\Lambda}{\mu}$ which is 49000000

6**.2 Numerical Simulation** The simulations were carried out using the parameters in table 1.1 for initial conditions. The final time was $t = 15$ years. We use Maple 16 software to do the analysis

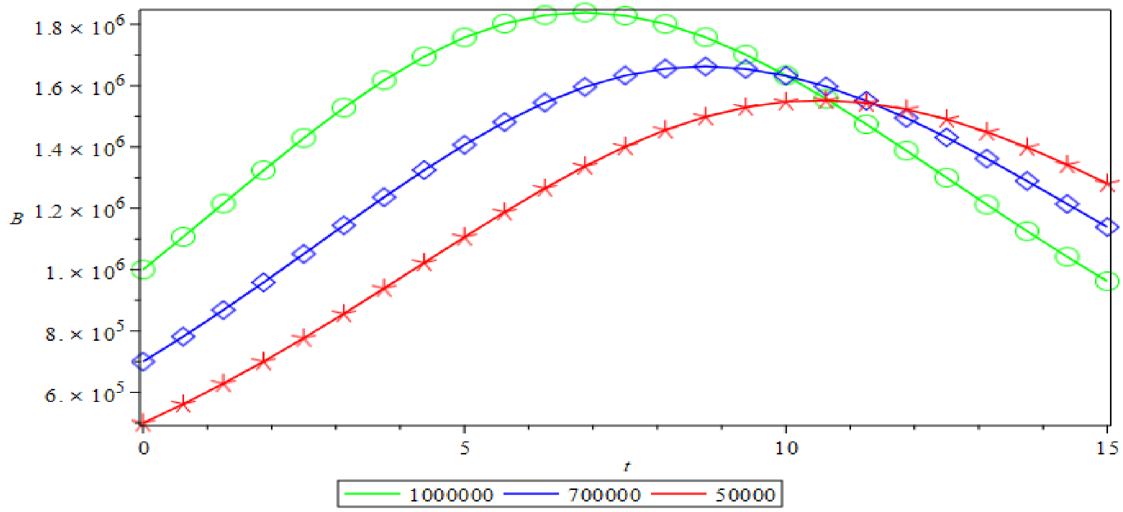


Figure 2: Total number of bandits with different initial variables conditions: $B(1)=1000000$ and $B(2)=700000$, $B(3)=500000$.. Control parameters used are as in Table 4.1 and Table 4.2 with $\varepsilon = j = \delta_3 = 0.25, \mu = 0.017$ which gives $R_C = 1.542$.

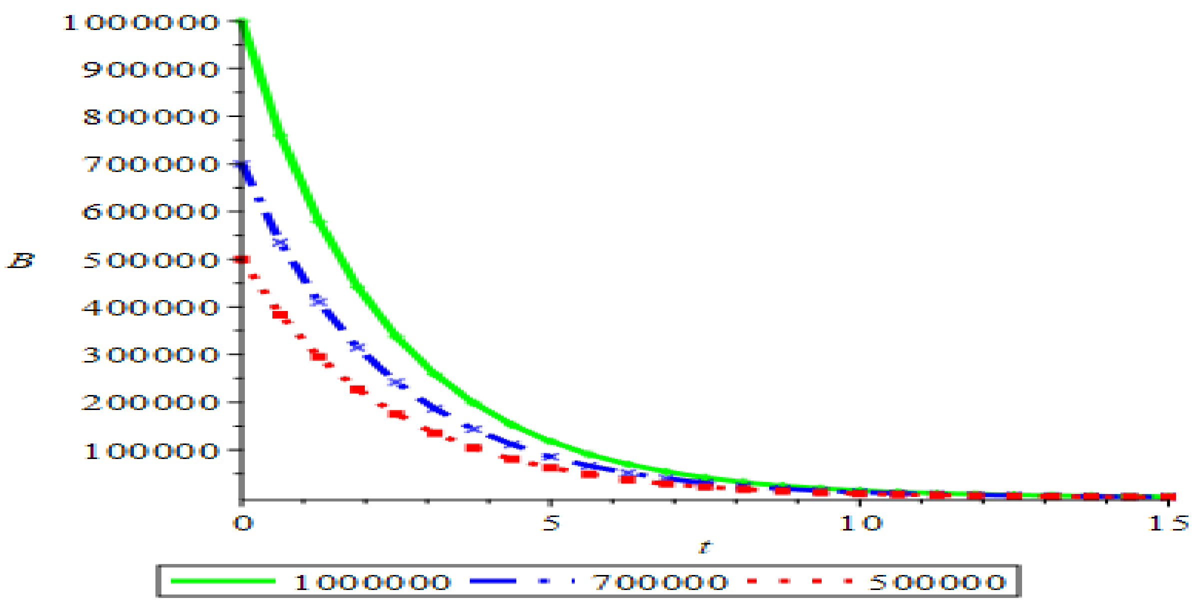


Figure 3: Total number of bandits with different initial variables conditions: $B(1)=1000000$ and $B(2)=700000$, and $B(3)=500000$, . Control parameters used are as in Table 4.1 and Table 4.2 with $\varepsilon = j = \delta_3 = 0.50$ $\beta = 0.9$, $\mu = 0.017$, and which gives $R_C = 0.589$.

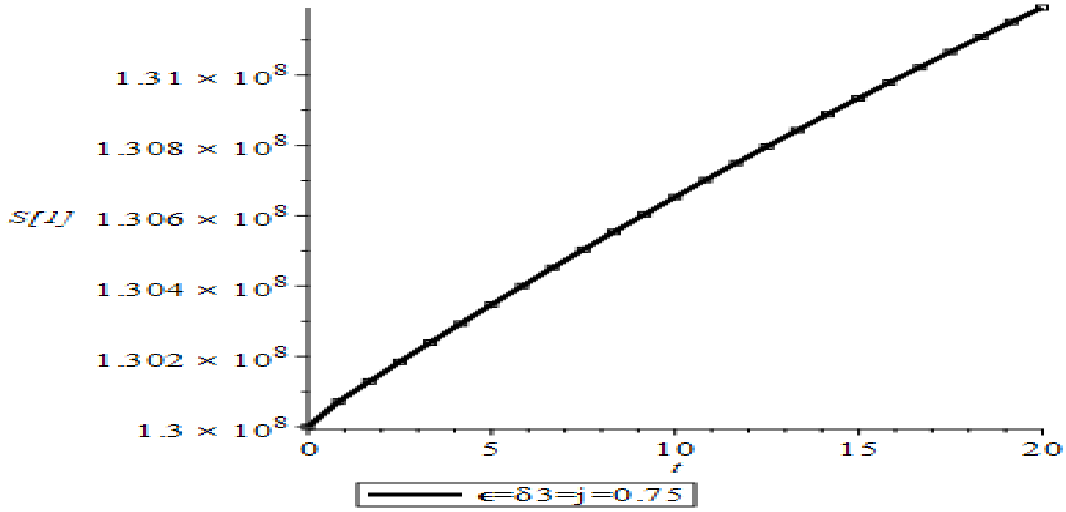


Figure 4: Total number of non-recruitable susceptible with different initial variables conditions: $B(1)=1000000$. Control parameters used are as in Table 4.1 and Table 4.2 with $\varepsilon=j=\delta_3=0.75\,\beta=0.9,\ \mu=0.017,$.

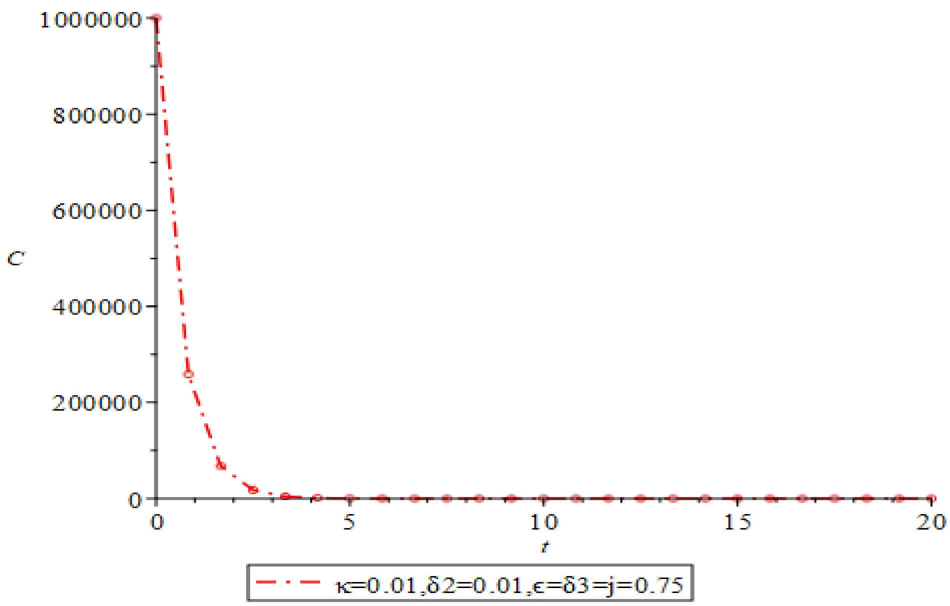


Figure 5: Total number of captives with different initial variables conditions: $B(1)=1000000$. Control parameters used are as in Table 4.1 and Table 4.2 with $\varepsilon=j=\delta_3=0.75\,\beta=0.9,\ \mu=0.017,$

Figure 6: Effect of effective reproduction number of 3 different control strategy levels. Parameter values used as in Table 4.2 with $\varepsilon=j=\delta_3=0.25.\varepsilon=j=\delta_3=0.5,\varepsilon=j=\delta_3=0$ on banditry

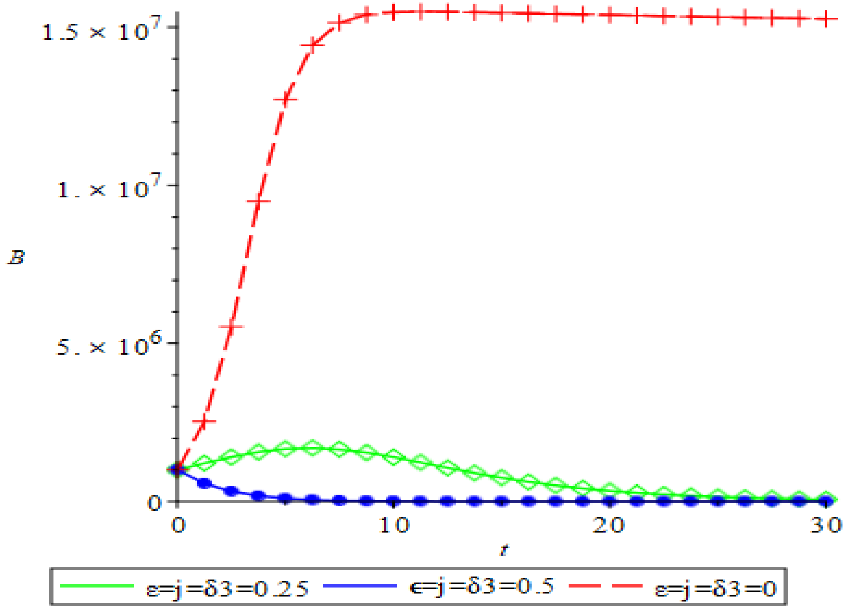


Figure 7: Effect of rehabilitation, elimination of bandits and preventing of recruitable susceptible (S_2) from joining bandits on banditry.

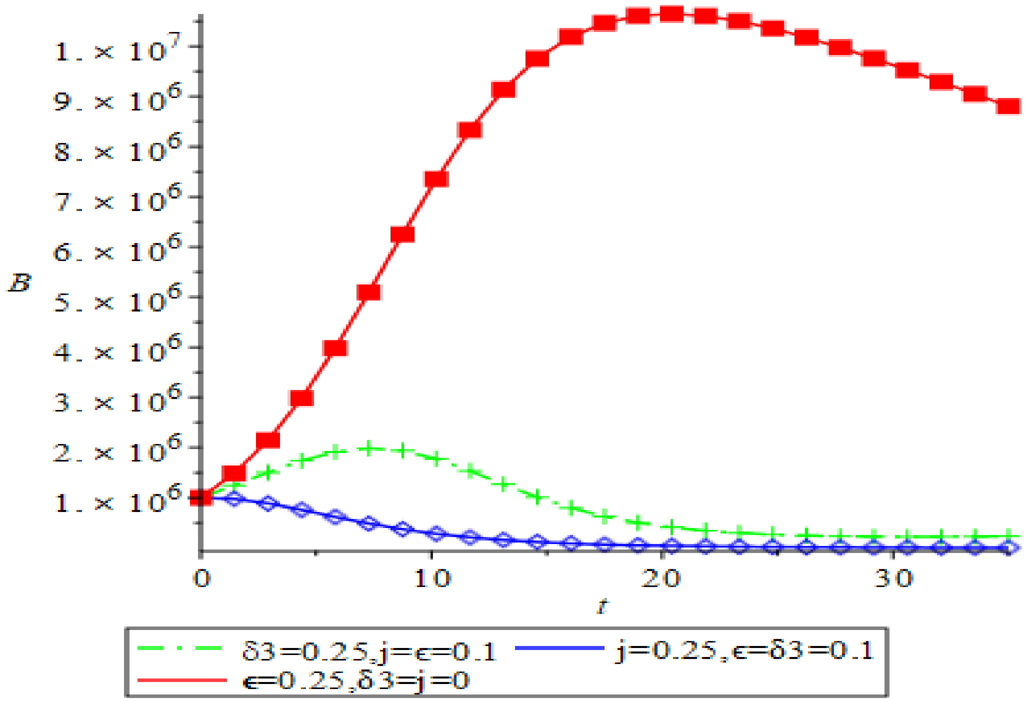
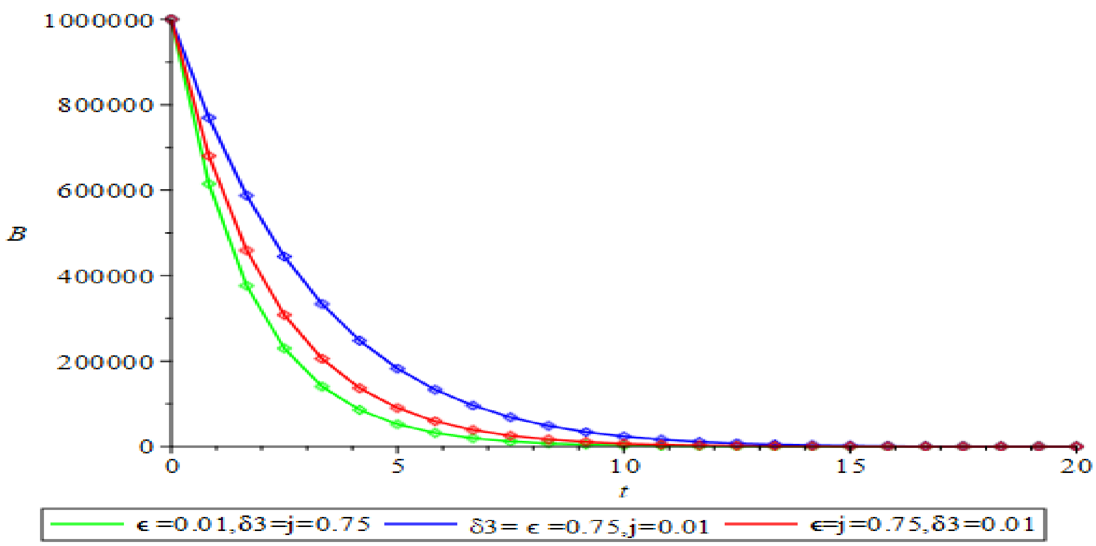


Figure 7: Graph of banditry against time comparing the effects on controls where we compare $j = \varepsilon = 0.1, \delta_3 = 0.75$ **and** $j = 0.75, \varepsilon = \delta_3 = 0.1$ **and finally** $\varepsilon = 0.75, \delta_3 = j = 0.1$

A suggested feasible solution as illustrated in Figure 4.*6 is:*

- Maintaining of 66% rehabilitating bandits in which majority of them will repent combined with 20% assumed killing of of those that either run away or attack military personnel and 12% goes to preventing S2 to join B.

Figure 8: Composite effects of killing of bandits by government/rehabilitating of bandits, elimination of bandits/preventing recruitable susceptible from joining banditry and preventing recruitable susceptible from joining banditry/rehabilitation of bandits.



6.3 Discussion of results

Figure 2 shows the global asymptotic stability of the endemic equilibrium, with the persistence of the solution profiles for $R_C = 1.542$ and different initial conditions for the variables as in Table 4.1 for hypothetical values. Similarly, with $R_C = 0.589$, using same variables as in Fig 3, we clearly see in Figure 3. that the solution profiles converges to the bandits free equilibrium in all cases. This confirms our analytical result of global asymptotical stability of bandits-free equilibrium.

figure 4, Its clearly observed that as soon as there is high control the Susceptible individuals S_1 with be increasing with respect to time. While in figure 5, It was observed that as soon as there is controls of capturing, eliminating and rehabilitation of bandits as well as effort of government in preventing S_2 to join banditry then banditry activities will not persist.

In figure 6, we observed that rehabilitation of kidnappers is best way to curve banditry activities followed by elimination of the bandits and then effort of government on preventing S_2 to join banditry.

Clearly, we observed from Figure 7, that capturing and rehabilitation as well as elimination of bandits have much impact than preventing recruitable S_2 to join banditry, why because if they are in rehabilitation, they may learn skills and end of repenting

Figure 8 is when we compare two controls against one it shows that elimination/rehabilitation of bandits is the best way to end banditry activities followed by killing/ ability of government in preventing S_2 to join Bandits.

6.4 Conclusion

This research focused on banditry activities considering government effort (Rehabilitation/ elimination of bandits) to target and end banditry activities. However, bandits have suffered multiple defeats as a result of intervention of government/military (joint task force) which includes force from Nigeria and part of Niger republic. The forces also received supports from federal government. The result of this research can further be extended to other field of knowledge to study the pattern of banditry in many geopolitical zones which pose a significant threat to the public, therefore necessary plans are made to address other issues in future research.

6.5 Recommendations

1. Its very difficult to tackle banditry activities without reducing the number of bandits by either elimination or rehabilitating them. Therefore government/military should check pattern in δ_3 or j of this work
2. Security agents should not restrict their selves in single state without blocking the boarders of neighboring states so that the bandits will not relocate to neighboring states.
3. Government should propose a practical tool for dealing with bandits, terrorist and kidnappers
4. Government strategy should not focus on bandits only but also prevent recruitable susceptible to join bandits by either enlighten or supporting them with skills that will engage them.
5. Government/ NGO should implement policies of employing youths so as to reduce the rate poverty and nonemployments in the society.

7 Conflict of Interest: On behalf of authors, I assure you there is no conflict of interest

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REFERENCES

[1] Abdurrahman, S. (2018). A Mathematical model for the transmission dynamics and control of Hepatitis B Virus. *Journal of science and technology and Mathematics Education.*

[2] Adamu, G., & Ibrahim, M. O. (2020). Mathematical modeling of dynamics behavior of terrorism and control. *Caspian Journal of Mathematical Sciences (CJMS) University of Mazandaran, Iran.* 9(1), 68-89.

[3] Akanni, J. O., & Abidemi, A. (2021). Mathematical analysis of the concomitant of illicit drug use & banditry in population. *Journal of Scientific and Engineering Research*, 5(8). Doi:10.21203/rs.3.rs-1089919/v1

[4] Momoh A.A. S. Musa, M.A. Alkali, A.M. Inalegwa, Mathematical Modeling and Optimal Control of Intervention Strategies for A Banditry Model, *International Journal of Science for Global Sustainability,* 9(2) (2023) 113{134. https://doi.org/10.57233/ijsgs.v9i2.467.

[5] Akanni J.O. A. Abidemi, Relationship between illicit drug users and bandits in a population: Mathematical modelling approach, Applied Mathematics, 17(3) (2023) 475{492. https://doi.org/10.18576/amis/170309.

[6] Akanni J.O. A. Abidemi (2021) Mathematical Analysis of the Concomitance of Illicit Drug Use and Banditry in a Population, 2021. https://doi.org/10.21203/rs. 3.rs-1089919/v1.

[7] Lawal J., A. Sule, A. Sani, Modeling and Optimal Control Analysis on Armed Banditry and Internal Security in Zamfara State, European Journal of Theoretical and Applied Sciences, 1(5) (2023) 1062{1075. https://doi.org/10.59324/ejtas.2023. 1(5).93.

[8] Aniayam, B., Okrinya, J., & Consul, I. (2018). Mathematical Modelling of Kidnapping with Rehabilitation. *Journal of Scientific and Engineering Research*, 5(8), 195-205.

[9] Diekmann. O., Heeisterbeek (1990) Construction of next-generation matrices for compartmental epidemic models

[10] Driessche V. den, & Watmough J. (2002) Reproduction numbers of infectious disease models. *Infectious disease modelling* 2(3)288-303

[11] Feng, Z., Huang, W. (2002), On the computation of reproduction number and its role on global stability. Scientific research

[12] Hoffman (2006) introduction to international terrorism. *United nation for drug and crime*, education for justice.counter-terrorism

[13] Hussain, S. Dynamical Behavior of Mathematical Model on the Network of Militants. *Punjab Uiversity Journal of mathematics, 51 (1), 51-60.*

[14] Julius, A., Makinde, A., & Ibrahim, M. O. (2021). Dynamics of Terrorism in Contemporary Society for Effective Management *Hindawi International Journal of Mathematics and Mathematical Sciences, 15,* ID 9056841.DOI.10.1155/2022/9056841

[15] Merkin. (1997) Introduction to theory of stability. Texts in Applied Mathematics volume 24.

[16] Ninuola I. A.(2017) Mathematical modeling and simulations. *2nd international workshop on mathematical modeling and simulations (IWMMAS,17).* University of Nsukka

[17] Okoye, C., Collins, O. C., & Mbah, G. C. E. (2020). Mathematical approach to the analysis of terrorism dynamics. *Security Journal*.

[18] Okrinya, A. B. (2018). A Mathematical Model on Kidnapping. *Journal of Engineering and Scientific Research*, Volume 5, No. 5, pages 102-110

[19] Okuonghae D., (2017). Mathematical analysis of epidemiological models for infectious disease. *Journal of applied mathematics,* Volume 12. No 10

[20] Olajide, J., & Abidemi, A. (2021). Mathematical Analysis of the Concomitance of Illicit Drug Use and Banditry in a Population.

[21] Rufai, M.A., (2021). **'**Im A bandit**':** Decade of Research on Armed Banditry in Zamfara State 15th University Seminar Presented at Usmanu Danfodiyo University, Sokoto Auditorium, Main Campus.

[22] Staffur, D. & Sahimi, M. (2007) Discrete simulation of the dynamics of opinion about extremism. *University of southern calfonia, les Angeles,* 90089-1211.

[23] Tasiu A.R., Shamsu S.M., Sammani. A. Hanafi J. and Muhammad A.(2024) " Mathematical Modelling of Kidnapping; Dynamics and Control" *Umyu Scientifica*, vol. 3 NO3,pp105-117DOI:10.56919/usci.2433.013

[24] Tasiu A.R., S. Sirajo A., Bala Y.I., Nasiru N.D., and A. Suleiman (2024) Dynamics of Banditry: Mathematical Modelling Approach *Bangmod international Journal of Mathematical and Computational Science* ISSN:2408-154X Vol 10 pp 77-96 doi:10.58715/bangmodjmcs.2024.10.6.